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13. Under this grant the researchers have primarily investigated two classes of eletromagnetic problems. The first contains the quantitative description of microwave heating of dispersive and conductive materials. Such problems arise, for example, when biological tissue are exposed, accidentally or purposefully, to microwave radiation. Other instances occur in ceramic processing, such as sintering and microwave assisted chemical vapor infiltration and other industrial drying processes, such as the curing of paints and concrete. The second class characterizes the scattering of microwaves by complex targets which possess two or more disperate length and/or time scales. Spatially complex scatters arise in a variety of applications, such as, large gratings and slowly changing guiding structures. The former are useful in developing microstrip energy couplers while the later can be used to model anatomical subsystems (e.g. the open guiding structure composed of two legs and the adjoining lower torso). Temporally complex targets occur in applications involving dispersive media whose relaxation times differ by orders of magnitude from thermal and/or electromagnetic time scales. For both cases the mathematical description of the problems, give rise to complicated ill-conditioned boundary value problems, whose accurate solutions require a blend of both asymptotic techniques, such as multiscale methods and matched asymptotic expansions, and numerical methods incorporating radiation boundary conditions, such as finite differences and finite elements.

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Research Activities

Under the current grant we have primarily investigated two classes of electromagnetic problems. The first contains the quantitative description of microwave heating of dispersive and conductive materials. Such problems arise, for example, when biological tissue are exposed, accidentally or purposefully, to microwave radiation. Other instances occur in ceramic processing, such as sintering and microwave assisted chemical vapor infiltration and other industrial drying processes, such as the curing of paints and concrete.

The second class characterizes the scattering of microwaves by complex targets which possess two or more disparate length and/or time scales. Spatially complex scatterers arise in a variety of applications, such as, large gratings and slowly changing guiding structures. The former are useful in developing microstrip energy couplers while the later can be used to model anatomical subsystems (e.g., the open guiding structure composed of two legs and the adjoining lower torso). Temporally complex targets occur in applications involving dispersive media whose relaxation times differ by orders of magnitude from thermal and/or electromagnetic time scales.

For both cases the mathematical description of the problems gives rise to complicated ill-conditioned boundary value problems, whose accurate solutions require a blend of both asymptotic techniques, such as multiscale methods and matched asymptotic expansions, and numerical methods incorporating radiation boundary conditions, such as finite differences and finite elements.

We also began some preliminary work on computing the shape of a three-dimensional target from time dependent scattering data. The applications of inverse problems of this type are numerous in both industrial and biological applications.

Finally, we began to develop efficient numerical algorithms for determining how incident pulses scatter from a variety of targets.

Before summarizing our progress we note that four Appendices are included with this report. Appendix A contains references that are pertinent to our research summarized below and are noted in the text by Roman Numerals. Appendix B contains a bibliography of papers published or submitted for publication that have been supported by this contract. Appendix C contains a list of presentations that have been made during the contract period. And Appendix D contains a list of recognitions obtained and other accomplishments made during this period.

1. Optimal Power Absorption in Lossy Targets.

When a time harmonic electromagnetic wave impinges upon a lossy target a portion of the incident wave is absorbed. This absorption is due to ohmic heating when the material is metallic, and is due to dipolar heating when the material is a dielectric. In either case the power absorbed by the target is given by

$$Q = \frac{\sigma}{2} \int_D |\mathbf{E}|^2 \, dx \, dy \, dz \tag{1}$$

where σ is the effective conductivity, $\mathbf{E}$ is the electric field within the target, and D is the compact region occupied by the scatterer. An important question to biomedical researchers interested in heating tissue and industrial researchers interested in material processing is the existence and computation of an optimal conductivity where the material heats most efficiently.

For small σ it is apparent from (1) and Maxwell's equations that

$$Q \sim \frac{\sigma}{2} \int_D |\mathbf{E}_0|^2 \, dx \, dy \, dz \tag{2}$$

where $\mathbf{E}_0$ is the electric field in the target when $\sigma = 0$. Thus, Q approaches zero linearly with σ. On the other hand as $\sigma \to \infty$, the electric field becomes exponentially small outside of a very thin boundary layer (a few skin depths in thickness) so that the absorbed power given by (1) approaches zero in this limit too [i-ii]. Since the functions appearing in (1) are differential with respect to σ, a maximum absorbed power Q^* occurs for an optimal $\sigma = \sigma^*$.

The functional dependence of σ^* and Q^* upon the various electrical and geometrical parameters of the target will be complicated and, in general, accessible only through extensive numerical scattering computations. In Reference 1 we have presented a simple approximate method which explicitly gives this relationship and significantly reduces the number of computations. We have chosen the problem of TM scattering from a finite, homogeneous, and lossy dielectric slab to demonstrate the method, because we can compare the exact results with our approximations.

Specifically, we have obtained large σ and small σ expansions of the solutions directly from Maxwell's equations and have computed the corresponding limits of (1). We have used a pade [iii] approximation to blend these results. This approximation has both limits built into it as well as a smooth transition between them. The validity, accuracy, and refinements of this approach, as well as its extension to higher dimensions, are under investigation.

We have recently begun the study of an analogous problem in which a Debye medium absorbs microwave energy. The idea is to blend the low frequency and high frequency expansions of the electric field together, using pade [iii] and Galerkin [iv] approximations, to obtain an approximation that is uniform over a large bandwidth of frequencies. Preliminary calculations on the pade approximation are quite encouraging and refinements of these approaches, as well as its extension to higher dimensions, are under investigation.

2. Microwave Heating of Ceramic Materials.

We have modeled and analyzed the heating of a ceramic slab [2,4] under TM-illumination in the small Biot number regime. The temperature distribution is almost spatially uniform in this limit and its evolution in time is governed by a first order nonlinear amplitude equation. This equation admits a time independent solution which is a multivalued function of the microwave power. The graph of this steady temperature as a function of the microwave power gives an S-shaped response curve when the electrical conductivity is modeled either as an exponential function of temperature or an Arrhenius law. The dynamics of the heating process are deduced, from the amplitude equation and the multivalued response, and are dependent upon the microwave power and initial conditions. For certain initial conditions and power levels the system evolves to the upper branch of the response curve which corresponds to thermal runaway. Other initial conditions and power levels force the system to evolve to the lower branch and a safe sintering temperature.

This heating process can be controlled in some sense by varying the microwave power in time at a rate commensurate with the thermal changes (v). Specifically, the power is allowed to changed in an exponential fashion from a higher to a lower power level. This relationship is turned into a differential equation which is appended to the amplitude equation to form an autonomous system of the second order. This system is analyzed using a phase-plane methods. The analysis shows the existence of a stable manifold which divides the phase plane into two parts: Trajectories in the region above this curve correspond to runaway heating while those below yield stable sintering. Various heating scenarios are presented and discussed in Reference 2.

We have considered a slightly more complicated control process [6] in which the rate of change of the microwave power in time is equal to the sum of two terms; the first is proportional to the power and the second to the temperature. When the second term is neglected the power is an exponential

function of time that increases from a given initial value to a final value which is chosen to be consistent with the sintering temperature [2,4]. For a wide value of parameters the analysis shows that the system does evolve to the sintering temperature. Unfortunately, the trajectory of the system passes through temperatures which are above the melting point of the material and this fact suggests, in the context of the one-dimensional model, that the sample is destroyed. However, when both terms are present in the control equation and they are properly chosen, the analysis shows that the sintering temperature can be reached without such an overshoot. Improper choices give rise to temperature overshoots and even relaxation oscillations.

3. Microwave Heating of Dispersive Media.

We have recently extended are one-dimensional work [8] to study the heating of a compact dispersive target by a pulsed, plane microwave. The dispersive character of the medium is described by a Debye model and the conductive nature is modeled by an ionic conductivity. The electrical and thermal parameters are allowed to depend upon temperature which gives rise to a highly nonlinear initial, boundary value problem. The governing equations are averaged in the limit as $\omega T_H \to \infty$ where T_H is a characteristic time at which heat diffuses in the target and ω is the microwave carrier frequency. A two-step algorithm is proposed for the numerical solution of these equations. When convection is weak, the algorithm converges very slowly. However, this problem is overcome by averaging the equations in the limit $T_H/T_B \to 0$ where T_B is a characteristic time describing energy loss by convection. This averaging yields a new theory from which a considerable amount of information can be deduced. Specifically, the temperature is spatially uniform in the target and evolves in time according to a first order, ordinary differential equation. The nonlinearity in this equation is a functional of the electric field within the target. This equation is solved for a number of specific examples and physical conclusions are drawn about certain heating processes. Finally, the problem of controlled heating is addressed where linear feedback is shown to be adequate in achieving a predetermined final temperature

4. Microwave Processing of Thin Ceramic Rods.

When a thin cylindrical ceramic sample is heated in a single mode waveguide applicator an interesting thermal phenomenon often occurs. A localized hot spot forms near the center of the sample and spreads outward from this point raising the entire ceramic to an elevated temperature (v-vi). This occurs even though the electric field intensity is essentially constant along the axis of the cylinder.

We have constructed a simple mathematical [9] model based upon the small Biot number limit which gives a plausible explanation of the formation, propagation, and growth of these hot-spots. Our model is simpler and easier to analyze than the one used in (v) because the variations of the temperature in the cross-section have been averaged out and this yields a reaction-diffusion equation in only one spatial variable. The dimensionless diffusion coefficient is given by

$$\epsilon = (a/L)^2 \frac{1}{B_i}$$

where a is the radius of the rod, L is its length, and B_i is the Biot number. This constant can be quite small especially when one is interested in sintering ceramic fibers. Thus, the problem reduces to a singularly perturbed reaction-diffusion equation of which much is known.

We have recently extended this work along two lines. In the first [14] we have allowed the thermal conductivity, the heat capacity, and the density of the ceramic to depend upon temperature. Although these dependencies are usually neglected when compared to the orders of magnitude change that occurs in the electrical conductivity, they are significant in affecting the velocity and ultimate character of the hot-spot dynamics. A complete description of this mechanism is contained in Reference 14.

We have also analyzed this heating problem when the sample is rotated 90^o in the cavity [16]. The model now takes the form of a nonlinear parabolic equation of reaction-diffusion type, with a spatially varying reaction term that corresponds to the spatial variation of the electromagnetic field strength in the waveguide. The equation is analyzed and a solution is found which develops a hot spot near the center of the cylindrical sample and which then propagates outwards until it stabilizes. The propagation and stabilization phenomenon concentrates the microwave energy in a localized region about the center where elevated temperatures may be desirable. Thus, we offer a plausible explanation for the experiments described in (vii) where localized fields are used to assist the joining of two ceramic rods.

Finally, we have modeled and analyzed the microwave heating of a carbon coated ceramic fiber [15,17]. Since the electrical conductivity of the carbon is much greater than that of the ceramic at room temperature, the coating rapidly heats and transfers thermal energy to the ceramic by conduction. This raises the average temperature of the material, on the convective time scale, and affects the initial condition of our small Biot theory. The results predicted by the theory qualitatively agree with experiments (viii) and give a new mechanism for reaching hitherto inaccessible portions of the s-shaped

response curve. A complete discription of this mechanism is contained in Reference 17.

5. Asymptotic/Numerical Hybrid Methods: An Example from Acoustic Scattering Theory.

We have developed a hybrid method [3], based upon the method of matched asymptotic expansions, to study the scattering of a plane acoustic wave from a large finite diffraction grating. The inner problem of an infinite grating is solved by a well-posed numerical method from which the modal amplitudes are determined. The outgoing scattered plane waves do not satisfy the Somerfeld condition so that this approximation becomes nonuniform in the far field. The outer problem of a finite strip of unknown sources is approximated by the method of stationary phase and the results are matched to those of the inner problem. The matching determines the amplitudes of the unknown sources. We have presented an explicit formula for the scattering cross section which blends both numerical and analytical results. It shows a highly localized scattering pattern whose maxima coincide with the Bragg angles of the infinite periodic structure. This localization as well as the magnitudes of the maxima clearly demonstrate the effects of the grating's finite size.

6. Large Finite Dielectric Gratings.

The problem of plane wave scattering by a finite, but electrically large, periodic dielectric grating on the interface between semi-infinite dielectric media is an important model of a microwave energy coupler. When the grating parameters fall in the resonance regime, i.e., the height and period of the grating are comparable to the wavelength of the incident wave, then numerical methods become inappropriate because of the disparate length scales present. We have applied the Method of Matched Asymptotic Expansions to construct a hybrid numerical/asymptotic method for efficiently analyzing this problem [18].

Appropriate scalings of the independent variables have resulted in inner and outer problems, whose respective solutions are obtained by a numerical and an analytical method, respectively. The approach has provided an alternative method to lengthy numerical calculations and has made the characterization of scattering by finite gratings whose length is of the order of tens of wavelengths. A closed form expression for the scattering pattern has been derived in terms of the Rayleigh reflection and transmission modal amplitudes which have been obtained from the numerical approach used to solve the inner problem. The details of these calculations and results are contained in Reference 18.

7. Large Membrane Array Scattering.

We have extended the above techniques to study the scattering of a plane acoustic wave from a large array of baffled membranes [10]. The structure is a simple model of an acoustic antenna. The far field pattern again is highly localized about the Bragg or mode angles of the inner representation. The maximum value of this function is directly proportional to the reflection coefficient determined numerically from the inner problem. Several numerical examples are presented illustrating these features. Moreover, it is observed that the asymptotic theory agrees quite well with the exact answer (obtained by using a finite difference scheme) when there are only three membranes present.

We have also developed a new numerical method for solving the inner problem (infinite grating) when many Bragg modes are present. The method is based upon a finite difference scheme used in conjunction with a new radiation boundary operator. This operator is local in character and effectively allows the Bragg modes to radiate out of the computational domain (fundamental cell). A full description of the method is given in Reference 13.

8. Acoustic Target Reconstruction Using Geometrical Optics Phase Information.

A simple algorithm, which uses the phase information from the geometrical optics limit, to construct the shape of an object has been presented in Reference 5. Essentially, the phase in the back scattered direction was shown to determine the equation of the tangent plane at the unique, but unknown, specular point. This plane depends upon the two spherical angles θ and ϕ, which describe the incident wave direction. The observation that the tangent plane envelopes the obstacle as θ and ϕ are varied, allowed the derivation of an explicit formula for the equation of the surface in terms of the measured scattered phase. An analogous two-dimensional formula has also been presented. A full description of the method is given in Reference 5.

9. Rapid Pulse Responses for Scattering Problems.

We have recently considered the numerical computation of the field scattered from a body in two dimensions due to an incident plane pressure pulse. In particular, we have examined the process of inferring the scattered field due to one incident pulse given the scattered field due to another incident pulse. The objective is to develop an indirect method that avoids the potentially expensive direct solution of the problem. Our approach is based on a formula expressing the scattered field as a convolution of a kernel with the incident pulse profile. The most straight forward generalization of this formula

to the discrete version of the scatter problem used in numerical computations does not allow the kernel to be inferred from a single numerical experiment—a difficulty we have called the *multi-source problem.* Preprocessing the incident pulses using simple interpolation formulas overcomes the multi-source problem giving an exact algorithm for computing the kernel. Selection of a sharp incident pulse (the Kronecker pulse) for the primary numerical experiment assures stability of this algorithm and permits extremely accurate prediction of the scattered fields for secondary incident pulses. A complete description of our results can be found in Reference 11.

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viii G.J. Vogt and W.P Unruh, "Microwave Hybrid Heating of Alumina Filaments", Microwaves: Theory and Applications in Material Processing II, Ceramic Transactions Vol. 36, The American Ceramic Society, Westerville, Ohio, 1993.

Appendix B: Papers published and submitted.

1. G. A. Kriegsmann, "Optimal Power Absorption in Lossy Targets", **Applied Mathematics Letters**, Vol. 4, N0. 4, 1991.
2. G. A. Kriegsmann, "Microwave Heating of Ceramics: A Mathematical Theory", in Microwaves: Theory and Applications in Materials Processing, ed., D.E. Clark, F.D. Gac, and W.H. Sutton, **Ceramic Transactions** 21, American Ceramic Society 1991, pp. 117-183.
3. G. A. Kriegsmann, "Asymptotic/Numerical Hybrid Methods: An Example from Acoustic Scattering Theory", **Proceedings of the Second Spanish Congress on Applied Mathematics**, Oviedo, Spain, 1991.
4. G. A. Kriegsmann, "Thermal Runaway in Microwave Heated Ceramics: A One-Dimensional Model", **Journal of Applied Physics**, Vol. 71, No. 4, 1992.
5. G. A. Kriegsmann, "Acoustic Target Reconstruction Using Geometrical Optics Phase Information", **I.M.A Journal on Applied Mathematics**, Vol. 48 , 1992.
6. G. A. Kriegsmann, "Thermal Runaway and Its Control in Microwave Heated Ceramics", **Proceedings of the 1992 Spring Materials Research Society Meeting**, San Francisco, 1992.
7. I. D. Abrahams, G. A. Kriegsmann, and E. L. Reiss, "On the Development and Control of Caustics Over Elastic Surfaces", **Journal of the Acoustical Society of America**, Vol. 92, 1992.
8. G. A. Kriegsmann, "Microwave Heating of Dispersive Media", **SIAM J. Appl. Math.**, VOl. 53, No.3, 1993.
9. G. A. Kriegsmann and P. Varatharajah, "Hot Spot Formation in Microwave Heated Ceramic Rods", in Microwaves: Theory and Applications in Materials Processing II, ed., D.E. Clark, F.D. Gac, and W.H. Sutton, **Ceramic Transactions** 23, American Ceramic Society 1993, in press.
10. G. A. Kriegsmann and C. L. Scandrett, "Large Membrane Array Scattering", **Journal of the Acoustical Society of America**, Vol. 93, 1993.
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13. G. A. Kriegsmann and C. L. Scandrett, "Decoupling Approximations Applied to Infinite Arrays of Baffled Membranes", **Journal of Computational Physics**, Vol. 111, No. 2, 1994.

14. G. A. Kriegsmann, "Growth and Stabilization of Hot Spots in Microwave Heated Ceramic Fibers", in Microwave Processing of Materials III, eds. R. Beatty, W. Sutton, and M. Iskander, Vol. 320, The Materials Research Society, Pittsburgh, 1994, in press.

15. G. A. Kriegsmann and B. A. Wagner, "Microwave Heating of Carbon Coated Ceramic Fibers", in Microwave Processing of Materials III, eds. R. Beatty, W. Sutton, and M. Iskander, Vol. 320, The Materials Research Society, Pittsburgh, 1994, in press.

16. M. R. Booty and G. A. Kriegsmann, "Microwave Heating and Joining of Ceramic Cylinders: A Mathematical Model", Methods and Applications of Analysis, submitted.

17. G. A. Kriegsmann and B. A. Wagner, "Microwave Heating of Carbon Coated Ceramic Fibers: A Mathematical Model", Journal of Applied Physics, submitted.

18. G. A. Kriegsmann and P. G. Petropoulos, "An Approximate Method for the Analysis of Finite Diffraction Gratings on Dielectric Interfaces", **IEEE Journal on Antennas and Propagation**, submitted.

Appendix C: Presentations.

1. "Microwave Heating of Ceramics", American Ceramics Society, Cincinnati, OH., April, 1991.

2. "A Study of Wave Interaction with a Slit Cylinder Using the OSRC Method", PIERS, Boston, MA., July, 1991.

3. "A Study of Wave Interaction with a Slit Cylinder Using the OSRC Method", IEEE/URSI, Toronto, July, 1991.

4. "Scattering from Complex Targets: A Model Problem", Second Span-

ish Congress on Applied Mathematics, Oviedo, Spain, September, 1991.

5. "The On Surface Radiation Condition", I.I.T., Department of Electrical Engineering, Chicago, November, 1991.

6. "Microwave Heating of Dispersive Media", Air Force School of Aerospace Medicine, San Antonio, TX, January, 1992.

7. "Controlled Microwave Heating of Materials", R.P.I., Department of Mathematics, March , 1992.

8. "Thermal Runaway and Its Control in Microwave Heated Ceramics", Spring Materials Research Society Meeting, San Francisco, May 1992.

9. "Controlled Microwave Heating", AFIT, Department of Mathematics, May 1992.

10. "Acoustic Scattering by Large Corrugated Surfaces: A Model Problem with Disparate Length Scales", Central University of Venezuela, Department of Mathematics, May, 1992.

11. "The On Surface Radiation Condition", Simon Bolivar University, Department of Mathematics, May, 1992.

12. "Hybrid Methods for Electromagnetic Scattering in Dispersive Media", Hanscom Air Force Base, Boston, June, 1992.

13. "Controlled Microwave Heating", University of Dundee, Scotland, June, 1992.

14. "Microwave Heating of Dispersive Media" SIAM meeting, L.A., July, 1992.

15. "Controlled Microwave Heating of Ceramics", AFOSR, Washington, D.C., August, 1992.

16. "Acoustic Propagation from a Point Source in a Shear Flow", ICASE, NASA Langley, September 1992.

17. "Acoustic Propagation from a Point Source in a Shear Flow", Department of Mathematics, SMU, Dallas, TX, September 1992.

18. "A Hybrid Method for Ultra-Short Pulse Electromagnetics", International Conference on Ultra-Wideband Short Pulse Electromagnetics, Polytechnical University, Brooklyn, NY, September 1992.